

Improving Perceptual Color Difference using Basic Color Terms

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Abstract

We suggest a new color distance based on two observations. First, perceptual color differences were designed to be used to compare very similar colors. They do not capture human perception for medium and large color differences well. Thresholding was proposed to solve the problem for large color differences, *i.e.* two totally different colors are always the same distance apart. We show that thresholding alone cannot improve medium color differences. We suggest to alleviate this problem using basic color terms. Second, when a color distance is used for edge detection, many small distances around the just noticeable difference may account for false edges. We suggest to reduce the effect of small distances.

1 Introduction

Color difference perception of just notably different colors and defining distances between very-similar colors has received considerable work [22, 29, 21, 35, 46]. In the CIE (International Commission on Illumination) community, distances of up to 7 CIELAB units, where 1 CIELAB unit approximately correspond to 1 just noticeable difference, are considered medium distances [17]. In this paper we refer to 0-7 CIELAB distances as very-similar as they capture just a small fraction of similar colors. See Fig. 1.

The CIEDE2000 color difference is considered the state of the art perceptual color difference [21, 47, 17]. CIEDE2000's recommended range for use is 0 to 5 CIELAB units [2]. The COM dataset was used to train and perceptually test the CIEDE2000 color difference. More than 95 percent of the distances between color pairs in the COM dataset are below 5 CIELAB units apart.

It was pointed out that the resulting color differences do not correspond well with human perception for medium to large distances. Rubner et al. [32] and Ruzon and

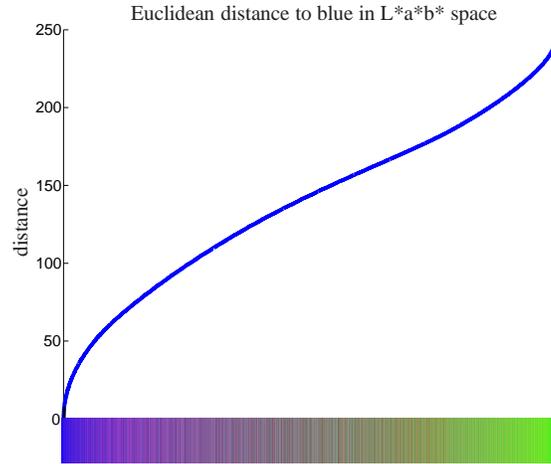


Figure 1: This figure should be viewed in color, preferably on a computer screen. The x-axis is colors image, where the colors are sorted by their Euclidean distance in $L^*a^*b^*$ space to the blue color. We can see that distances up to 7 CIELAB units capture just a small fraction of similar colors (zoom in to see in the left a black line touching the y-axis in 7 CIELAB units).

Tomasi [33] used a negative exponent on the color difference. Namely, all totally different colors are essentially assigned the same large distance. Pele and Werman [27] noted that a negative exponent changes the values in the small distance range. Pele and Werman [27] and Rubner et al. [32] observed a reduction in performance due to this change. Pele and Werman suggested thresholding the color difference as it does not change the small distances. Thresholding color distances is justified by the fact that if people are directly asked for a judgment of the dissimilarity of colors far apart in color space, subjects typically find themselves unable to express a more precise answer than “totally different” [18]. An additional advantage of thresholding color distances is that it allows fast computation of cross-bin distances such as the Earth Mover’s Distance [27] or using the transformation to a similarity measure of one minus the distance divided by the threshold, the Quadratic-Chi [28].

This paper shows that CIEDE2000 is not a good distance for the medium range and using any monotonic function of CIEDE2000 (including a thresholding function) cannot solve the problem. For example, using a thresholding function we cannot make DarkSkyBlue be more similar to Blue than to HotPink. See Fig. 2 for more examples.

We suggest an improvement based on basic color terms. Specifically we use Berlin and Kay’s eleven English basic color terms[6]. However, the generalization to other color terms is straight forward. We suggest adding to color differences the distance between their basic color terms probability vectors. As basic color terms are correlated (*e.g.* red and orange) we suggest using a cross-bin distance for these probability vectors. That is, a distance which takes the relationships between bins (each bin represents a basic color term) into account. Specifically we use the Earth Mover’s Distance [32]

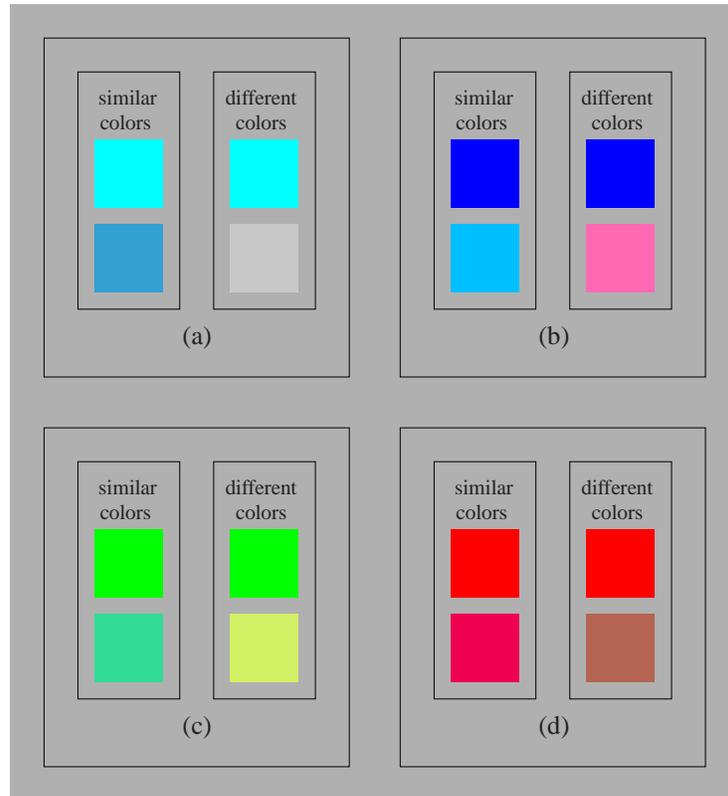


Figure 2: This figure should be viewed in color, preferably on a computer screen. Each sub-figure (a)-(d) contains a pair of similar colors and a pair of different colors. In all of these examples, the CIEDE2000 distance between the visually similar colors is higher than the distance between the different colors. Our proposed distance succeeds in all of these examples.

as it was used successfully in many applications (*e.g.* [32, 31, 33, 27] and references within).

The probability vectors are obtained with the color naming method developed by van de Weijer et al. [44]. Other methods for color naming such as [9, 19, 34, 3, 16, 26, 4, 24, 23, 5] can also be used. We chose van de Weijer et al. method as it has excellent performance on real-world images and as the code for it is publicly available. However, CIEDE2000 was learned under calibrated conditions, while van de Weijer et al. method was learned from natural images. Thus, other color naming methods might produce better results. This is left for future work.

Our proposed solution is not equivalent to increasing the weight of the hue component in the color difference. Color names are not equivalent to hue. For example, although a rainbow spans a continuous spectrum of colors, people see in it distinct bands which correspond to basic color terms: red, orange, yellow, green, blue and pur-

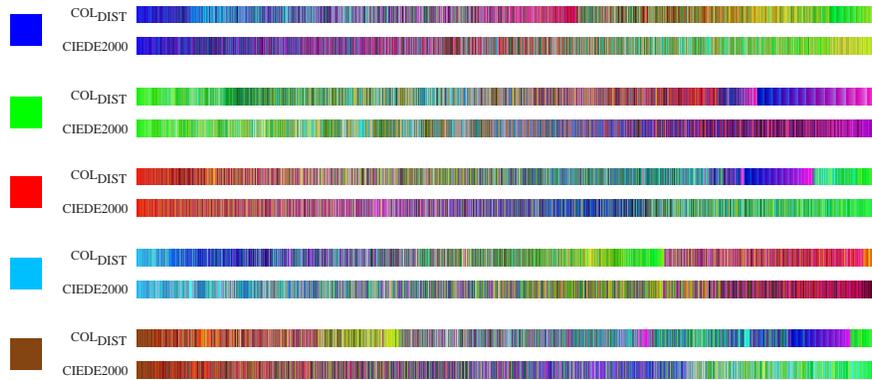


Figure 3: This figure should be viewed in color, preferably on a computer screen. Use the pdf viewer’s zoom to see the colors. We show colors, sorted by their distance to the color on the left. COL_{DIST} is our new perceptual color difference. Several observations can be derived from these graphs. First, our distance is perceptually better in the medium distance range. Note that the group of similar colors (left side of each color legend) is more similar to the color on the left using our distance. For example, in the top light blues are similar to blue, while using CIEDE2000 they are very different (thus appear on the right). It should be noted that our distance uses a sigmoid function, so that very similar colors on the left are essentially assigned the same small distance and totally different colors on the right are essentially assigned the same large distance. Finally, although our distance is perceptually more meaningful, it is still far from being perfect.

ple. In addition, some basic color terms are not different in their hue component. *e.g.* achromatic colors such as white, gray and black or orange and brown which shares the same hue.

A second problem that occurs when using color differences for edge detection is that many small distances around the just noticeable difference may account for false edges. We suggest to use a sigmoid function to reduce the small distances effect. As we mentioned before, using a negative exponent function in order to assign to all totally different color pairs the same distance reduced performance [32, 27]. We explain this by the fact that a negative exponent is a concave function. We show that a convex function should be applied to small differences.

We present experimental results for color edge detection. We show that by using our new color difference the results are perceptually more meaningful.

Our solution is just the first step of designing a perceptual color difference for the full range of distances. Our major contribution is raising the problem of current state-of-the-art color differences in the small and medium distance range.

This paper is organized as follows. Section 2 is an overview of related work. Section 3 introduces the new color difference. Section 4 presents the results. Finally, conclusions are drawn in Section 5.

2 Related Work

MacAdam’s [22] pioneering work on chromaticity discrimination ellipses, which measured human perception of just noticeable differences led the way to the development of the $L^*a^*b^*$ space [29] which is considered perceptually uniform; *i.e.* for very-similar colors, the Euclidean distance in the $L^*a^*b^*$ space corresponds to the human perception of color difference well. Luo et al. [21] developed the CIEDE2000 color difference which is now considered the state of the art perceptual color difference [21, 47, 17].

Although color is commonly experienced as an indispensable quality in describing the world around us, state-of-the-art computer vision methods are mostly based on shape description and ignore color information. Recently this has changed with the introduction of new color descriptors [39, 14, 25, 40, 42, 8, 36]. However, although color is a point-wise property (*e.g.* bananas are yellow), most of these features capture geometric relations such as color edges.

Wertheimer [1] suggested that among perceptual stimuli there are “ideal types” that are anchor points for perception. Rosch [30] proposed that in certain perceptual domains, such as color, salient prototypes develop non arbitrarily. An influential paper by Berlin and Kay [6] defined basic colors as color names in a language which are applied to diverse classes of objects and whose meaning is not subsumable under one of the other basic color names and which are used consistently and by consensus by most of the speakers of the language. In their pioneering anthropological study, they found that color was usually partitioned into a maximum of eleven basic color categories of which three were achromatic (black, white, grey) and eight chromatic (red, green, yellow, blue, purple, orange, pink and brown). This partitioning was a universal tendency to group color around specific focal points as was conjectured by Wertheimer [1] and Rosch [30].

Considerable work has been carried out in the field of computational color naming, see *e.g.* [9, 19, 34, 3, 16, 26, 4, 24, 23, 5, 44] and references within. Recently van de Weijer et al. [44] presented a new color naming method based on real-world images. The color names are Berlin and Kay’s [6] eleven English basic color terms. Van de Weijer and Schmid [43] showed that a color description based on these color names outperforms descriptions based on photometric invariants. The explanation is that photometric invariance reduces the discriminative power of the descriptor.

Inspired by van de Weijer and Schmid’s work we suggest using the basic color names to correct the state-of-the-art color difference, CIEDE2000 in the medium distance range.

3 COL_{DIST} : The New Color Difference

Given two colors $C^1 = [R^1, G^1, B^1]$ and $C^2 = [R^2, G^2, B^2]$, we first convert¹ them into $L^*a^*b^*$: $S^1 = [L^1, a^1, b^1]$ and $S^2 = [L^2, a^2, b^2]$. Second, we compute the basic color term probability vectors: P^1, P^2 ; where P_i^n is the probability that the color C^n

¹We used Matlab’s default conversion which uses CIE illuminant D50 as the reference illuminant, known as the white point, which is also the default illuminant specified in the International Color Consortium specifications.

is the basic color term i (*i.e.* black, blue, brown, grey, green, orange, pink, purple, red, white or yellow). These probability vectors are computed using the van de Weijer et al. color naming method [44]. Now each color C^n is represented by an 14-dimensional vector: $V^n = [S^n, P^n] = [L^n, a^n, b^n, P_1^n, \dots, P_{11}^n]$. The distance between the two colors (parameterized with T , D , α and Z) is defined as:

$$d_1(S^1, S^2) = \frac{\min(\text{CIEDE2000}(S^1, S^2), T)}{T} \quad (1)$$

$$d_2(P^1, P^2) = \text{EMD}(P^1, P^2, D) \quad (2)$$

$$d_3(V^1, V^2) = \alpha d_1 + (1 - \alpha) d_2 \quad (3)$$

$$\text{COL}_{\text{DIST}}(V^1, V^2) = \frac{1}{1 + e^{-(Zd_3 - \frac{Z}{2})}} \quad (4)$$

In Eq. 1, d_1 is a thresholded and scaled CIEDE2000 color difference. We threshold it as it is recommended for use only for small distances [2]. We used $T = 20$ as was used in Pele and Werman [27]. We divide by T so that d_1 is between 0 and 1.

In Eq. 2 d_2 is the distance between the two basic colors probability vectors. As the bins in the eleven basic colors probability vectors are correlated (*e.g.* orange and red), we use the Earth Mover’s Distance that takes this correlation into account. The correlation is encoded in D which is an 11×11 matrix, where D_{ij} is the distance between basic color term i to basic color term j . We estimated D using the joint distribution of the basic color terms. That is, given the matrix M of all probability vectors for the colors in the RGB cube ($2^{15} \times 11$ matrix, as each dimension of the RGB cube was quantized with jumps of 8 [44]) we define $D = D_{ij}$ as:

$$\hat{D}_{ij} = 1 - 2 \left(\frac{\sum_n \min(M_{ni}, M_{nj})}{\sum_n M_{ni} + M_{nj}} \right) \quad (5)$$

$$D_{ij} = \frac{\min(\hat{D}_{ij}, t)}{t} \quad (6)$$

We threshold \hat{D}_{ij} as EMD is recommended to be used with thresholded ground distances [27]. We used the threshold $t = 0.7$. Finally we scale it so that $0 \leq D_{ij} \leq 1$ (which implies $0 \leq d_2 \leq 1$). The resulting matrix (see Fig. 4) is perceptually plausible; *i.e.* similar basic color terms are: grey and white, grey and black, orange and red, etc.

The Earth Mover’s Distance (EMD) [32] is defined as the minimal cost that must be paid to transform one histogram into another, where there is a “ground distance” (that is, the matrix D) between the basic features that are aggregated into the histogram. Here the basic features are the eleven English basic color terms. The formula for the EMD between the two probability vectors P^1 and P^2 is defined as²:

²This is a simplification of the original definition for the case where the histograms are probability vectors.

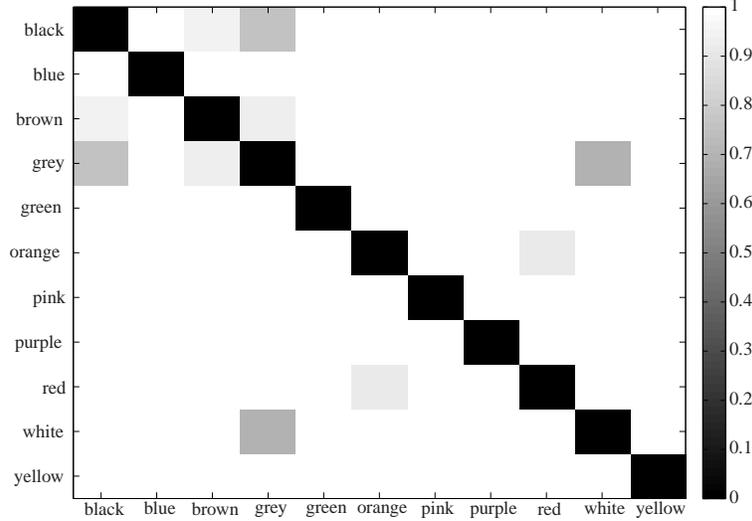


Figure 4: The learned basic color terms ground distance matrix.

$$\begin{aligned}
 \text{EMD}(P^1, P^2, D) &= \min_{\{F_{ij}\}} \sum_{i,j} F_{ij} D_{ij} \quad s.t \\
 \sum_j F_{ij} &= P_i^1, \quad \sum_i F_{ij} = P_j^2, \\
 \sum_{i,j} F_{ij} &= 1, \quad F_{ij} \geq 0
 \end{aligned} \tag{7}$$

In Eq. 3, d_3 is a linear combination of d_1 and d_2 . We used $\alpha = \frac{1}{2}$.

Finally, in Eq. 4, the distance is scaled so that it is in the range $[-\frac{Z}{2}, \frac{Z}{2}]$ (we used $Z = 10$) and then the logistic function (a sigmoid function) is finally applied. The sigmoid function reduces the effect of small distances and essentially gives all totally different colors the same distance.

4 Results

In this section we present color edge detection results. We used Ruzon and Tomasi generalized compass edge detector [33]. We used this method for two reasons. First, the code is publicly available. Second, the code uses only color cues for the edge detection which enables us to isolate color difference performance.

Ruzon and Tomasi's method [33] divides a circular window around each pixels in half with a line segment. Then it computes a sparse color histogram (coined signature

in their paper) for each half and computes the Earth Mover’s Distance (EMD) [32] between the two histograms. The EMD uses a ground distance matrix D between the colors. Ruzon and Tomasi converted the images to $L^*a^*b^*$ and then used a negative exponent of the Euclidean distance as the ground distance between colors:

$$d_e(S^1, S^2) = 1 - e^{\frac{-\| [L^1, a^1, b^1] - [L^2, a^2, b^2] \|_2}{\gamma}} \quad (8)$$

Ruzon and Tomasi used $\gamma = 14$ in their experiments. We compare the edge detection results using this distance to our proposed COL_{DIST} . We compared also to d_1 which is a thresholded CIEDE2000 distance as was used by Pele and Werman for image retrieval [27]. We also tried our proposed COL_{DIST} without the sigmoid function or without the color correction ($\alpha = 1$) or without the CIEDE2000 term ($\alpha = 0$) but the results using COL_{DIST} were the best. Results are presented in Figs. 5,6,7,8. The results show that the new color difference is able to detect color edges much better than the state of the art. The resulting edge maps are much cleaner. See figures captions for more details.

5 Conclusions

We presented a new color difference - COL_{DIST} and showed that it is perceptually more meaningful than the state of the art color difference - CIEDE2000. We believe that this is just the first step in designing perceptual color differences which perform well in the medium range.

It is easy to generalize our method to other color name sets (such as the Russian which separates blue into *goluboi* and *sinii*). All one needs to do is to calculate the ground distance between all color terms. This can be done by using the joint distribution of the new set of color terms. In future work it will be interesting to check other color naming methods such as [9, 19, 34, 3, 16, 26, 4, 24, 23, 5]

A major difficulty of analyzing color images is the illumination variability of scenes. Color invariants are often used to overcome this problem. However, Van de Weijer et al. [44, 43] showed that invariants are not discriminative enough. For example, invariants usually do not distinguish between achromatic colors (black, gray and white). Using color constancy or partial normalization algorithms [12, 13, 15, 41, 11, 10, 20, 7, 37] which do not necessarily reduce all distinctiveness may partially alleviate this problem. This method was used to improve color naming by Benavente et al. [3].

Color perception is also affected by spatial and texture cues. It will be interesting to combine COL_{DIST} with spatial and texture models [48, 45, 38]

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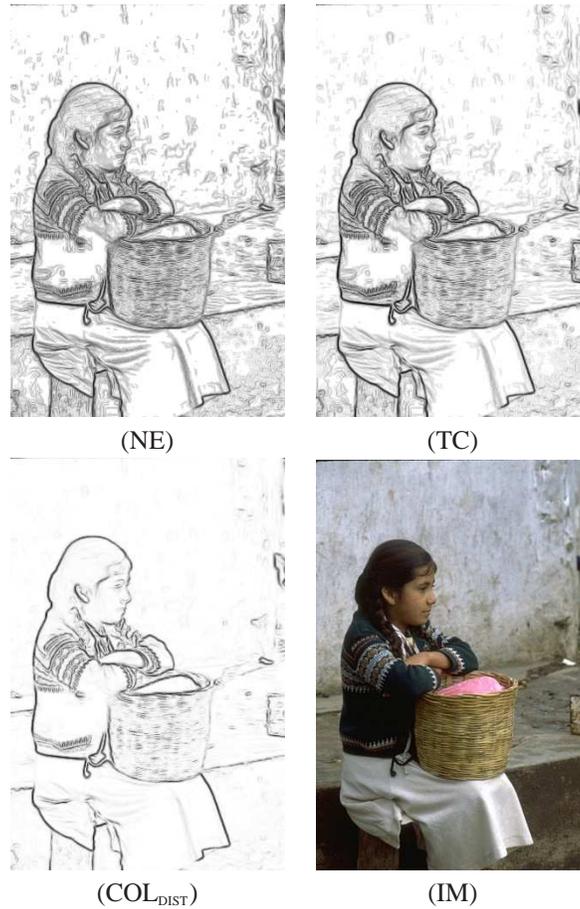


Figure 5: Edge detection with the generalized compass edge detection [33] using the following color differences: (NE) A negative exponent applied on the Euclidean distance in $L^*a^*b^*$ space (used in [33]). (TC) A thresholded CIEDE2000 distance (used in [27] for image retrieval). See Eq. 1. (COL_{DIST}) Our proposed COL_{DIST} . (IM) The original image.

Our result is much cleaner. Note that our method detects the right boundary of the basket without detecting many false edges, while in (NE) and (TC) the false edges magnitude is larger than the right boundary of the basket.

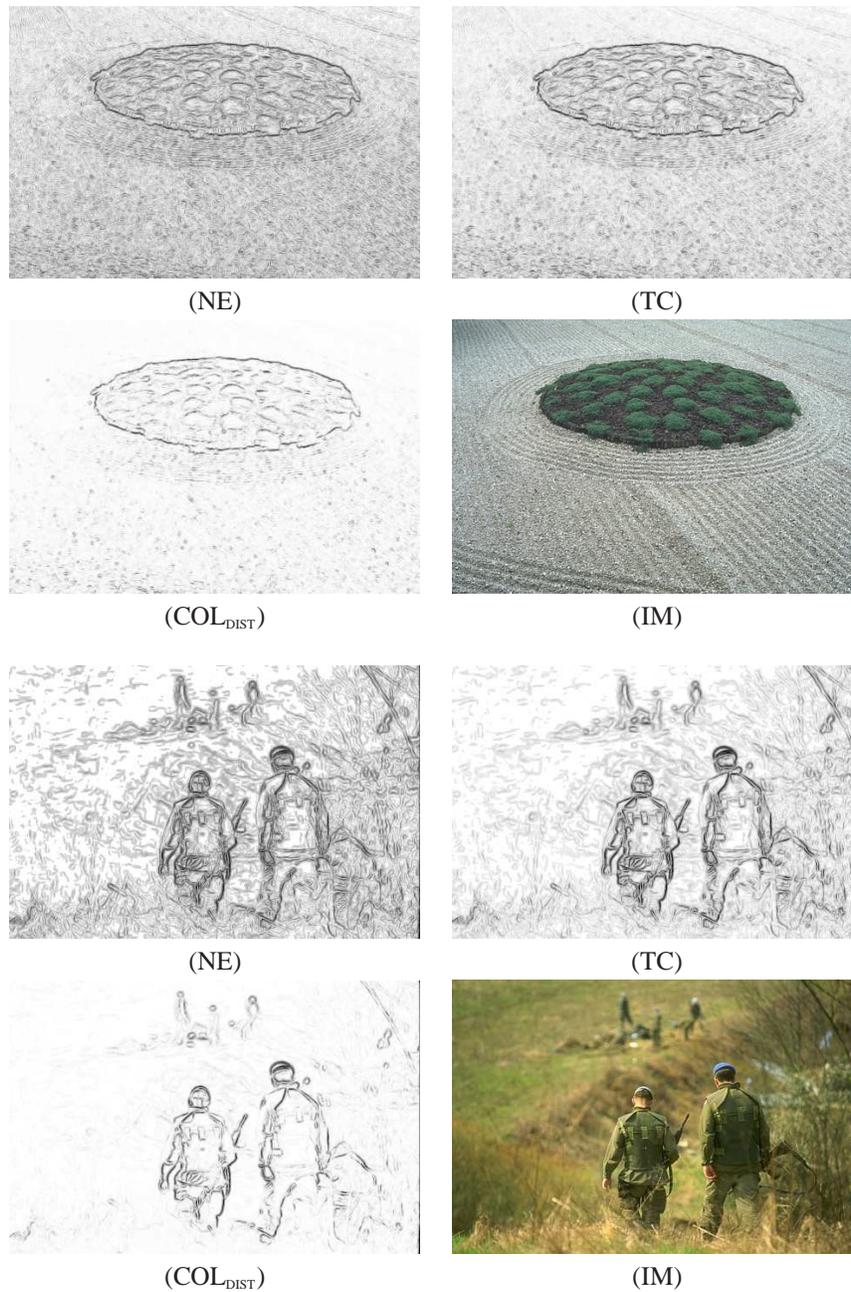


Figure 6: Edge detection with the generalized compass edge detection [33] using the following color differences: (NE) A negative exponent applied on the Euclidean distance in $L^*a^*b^*$ space (used in [33]). (TC) A thresholded CIEDE2000 distance (used in [27] for image retrieval). See Eq. 1. (COL_{DIST}) Our proposed COL_{DIST}. (IM) The original image.

Our results are much cleaner. Note that on the top the clean detection of the bushes boundaries.

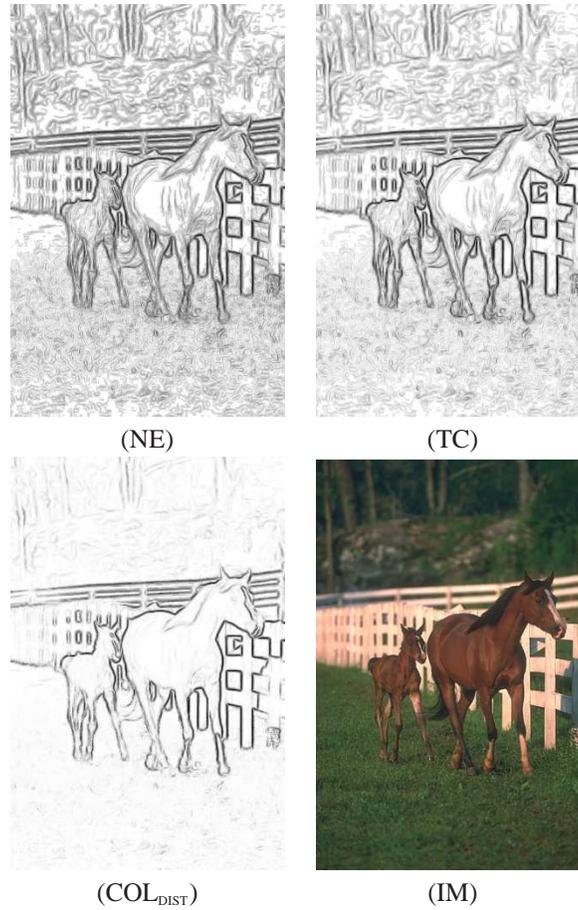


Figure 7: Edge detection with the generalized compass edge detection [33] using the following color differences: (NE) A negative exponent applied on the Euclidean distance in $L^*a^*b^*$ space (used in [33]). (TC) A thresholded CIEDE2000 distance (used in [27] for image retrieval). See Eq. 1. (COL_{DIST}) Our proposed COL_{DIST}. (IM) The original image. Our results are much cleaner.

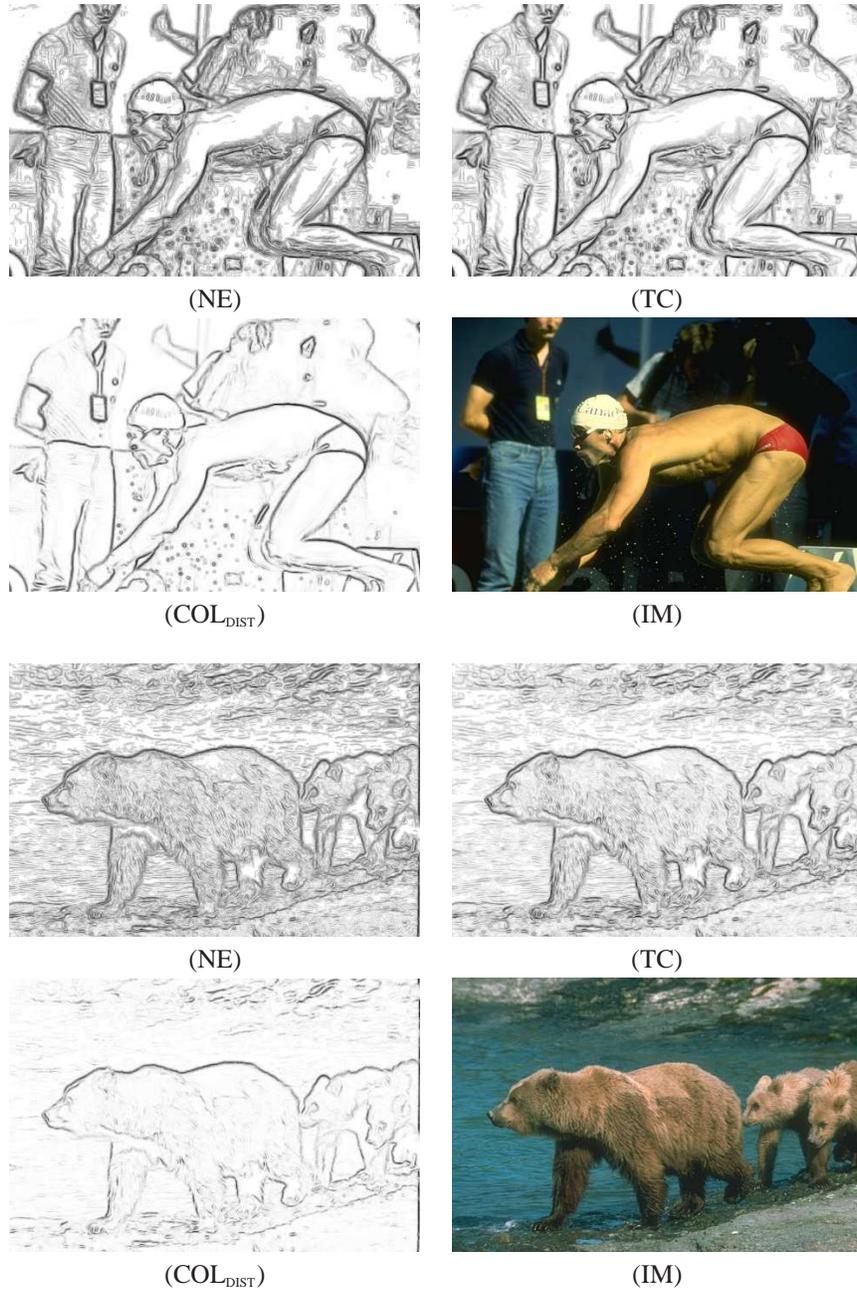


Figure 8: Edge detection with the generalized compass edge detection [33] using the following color differences: (NE) A negative exponent applied on the Euclidean distance in $L^*a^*b^*$ space (used in [33]). (TC) A thresholded CIEDE2000 distance (used in [27] for image retrieval). See Eq. 1. (COL_{DIST}) Our proposed COL_{DIST}. (IM) The original image.

Our results are much cleaner. Note the strong responses on the fur of the bear and in the left person on the top T-shirt using (NE) and (TC). Note that the spots around the swimmer in all methods are due to successful detection of the water drops.

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